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HEAVY QUARK EFFECTIVE THEORY *

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After a brief introduction, the results of application of the heavy quark effective theory to semileptonic decays of B mesons are discussed. A nearly-model-independent extraction of $|V_{cb}|$ from data is described. Application of the effective theory to inclusive semileptonic decays of heavy mesons, to semileptonic decays of heavy baryons, and to the nonleptonic decays of hadrons are outlined. We conclude by mentioning other areas of research in this very active field.

I. PRELIMINARIES

One of the main thrusts of experimental particle physics at present is the determination of the parameters of the standard model, as precision measurements of these parameters are essential for our understanding of this model, and for identification of possible new physics. Among these parameters, the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are very important for our understanding of the origin of CP violation, for instance. These matrix elements may be measured in processes in which the charged currents of the electroweak interaction come into play.

The most problematic aspect of these measurements results from the fact that, while we are interested in the electroweak transitions between quarks of different flavors, what we observe are the transitions between hadrons containing these quarks. We know that the theory that describes the binding of quarks within hadrons is quantum chromodynamics (QCD). We also know that, at the energy scales in which we are interested, namely the energy scales appropriate for binding quarks in hadrons, the coupling constant of QCD is large, so that any perturbative treatment is invalid. Unfortunately, we do not yet know how to construct a non-perturbative treatment of QCD.

The implication is that the extraction of the CKM matrix elements from experimental measurements is difficult. Until recently, the most common means of extracting these elements relied on the construction of specific models for the form factors describing the processes of interest. This necessarily introduced some model-dependence in the values extracted. With the advent of the heavy quark effective theory, the situation has improved, somewhat.

The heavy quark effective theory (HQET) [1–4] is an effective theory that arises in a specific limit of QCD. This limit is that in which the mass of one of the quarks in a hadron is taken formally to infinity. In this limit, two new symmetries, above and beyond those usually associated with QCD, arise. These two symmetries are a spin symmetry and a flavor symmetry. It is very important to emphasize that these are symmetries of the *light* component of the hadron, the so-called brown muck. In the formal limit, the heavy quark inside the hadron acts as an essentially static source of color, moving with fixed four-velocity.

The spin symmetry means that the properties of the brown muck are independent of the spin orientation of the heavy quark. This is very much analogous to the situation in quantum electrodynamics (QED) in the hydrogen atom, where the electronic spectrum (the spectrum of the *light* component) is largely independent of the spin orientation of the nucleus, the proton, say. Flipping this spin leads to the well known 21 cm line in the hydrogen spectrum, and corresponds to an energy of $\simeq 10^{-6}$ eV, compared with the typical spectral energy of $\simeq 1$ eV.

One immediate consequence of the spin symmetry for QCD is the existence of degenerate multiplets of states. For brown muck with spin j, there are two possible hadronic states with $J = j \pm 1/2$. At leading order in HQET, these two sets of states are degenerate. Thus, the heavy pseudoscalar mesons (B, D) are degenerate with their respective vector counterparts (B^*, D^*), as are the baryons Σ_Q and Σ_Q^* ($\Sigma_Q \equiv [(qq)_1 Q]_{1/2}$, $\Sigma_Q^* \equiv [(qq)_1 Q]_{3/2}$). Furthermore, the splitting between these pairs of states can only arise at order $1/m_Q$. This has long been thought to be a quark-model result (arising from one-gluon exchange, for example). Here we see that it is a QCD result, with no input from models. In addition, the spin-1/2 Λ_Q ($\Lambda_Q \equiv [(qq)_0 Q]_{1/2}$) is special, as the spin is carried exclusively by the heavy quark, and this state is alone in its heavy multiplet.

The flavor symmetry implies that the properties of the brown muck are independent of the flavor of the heavy quark. Again, the analogy one may use here is that of the hydrogen atom, in which the electronic spectrum is largely independent of whether the nucleus is a proton, a deuteron, a triton or even a muon. For QCD, this symmetry implies that the spectra of hadrons containing b and c quarks should be very similar.

Of course, in the real world, we do not have quarks that are infinitely heavy. We may choose to treat the b and c quarks as heavy. However, it is clear that we must take finite mass effects into account, especially for the c quark. HQET provides the framework for doing this systematically. It also allows us to include QCD radiative corrections in a systematic way.

II. A LITTLE FORMALISM

A. The Lagrangian

Consider a heavy quark Q with mass m_Q. In order to construct an effective theory in the formal limit $m_Q \to \infty$, we must redefine quantities that contain

explicit reference to the mass of the quark. In particular, quantities that scale with the mass of the quark will become formally infinite. We begin with the momentum of the quark P_Q, which we write as

$$P_Q^\mu = m_Q v^\mu + k^\mu. \tag{1}$$

v^μ is the four-velocity of the quark, which is the same as that of the hadron. Since the velocity has no reference to the quark's mass, it is the kinematic variable of choice in the effective theory that we construct. The first term of eqn. (1) clearly scales with the quark's mass, while the second term does not. This latter term is a measure of how far off its mass shell the quark is, and is the result of interaction with the brown muck of the hadron. It is also of the order of the energy scale appropriate to the brown muck, which is a few hundred MeV. Note that since $m_Q \to \infty$, a change in the velocity of the heavy quark requires an infinite momentum change. Thus, heavy quarks of different velocities do not communicate, leading to what is termed a 'velocity superselection rule' [2].

The part of the QCD Lagrangian that describes the quark field is

$$\mathcal{L}_Q = \bar{Q}\,(i\not{D} - m_Q)\,Q. \tag{2}$$

We define a new heavy quark field

$$h_v^{(Q)}(x) = h_v^{+(Q)}(x) + h_v^{-(Q)}(x),\;\; h_v^{\pm(Q)}(x) = \left(\frac{1 \pm \not{v}}{2}\right)\exp\left(\pm i m_Q v\cdot x\right)Q(x), \tag{3}$$

in terms of which the Lagrangian above becomes

$$\mathcal{L}_v^{(Q)} = i\bar{h}_v^{+(Q)} v\cdot D h_v^{+(Q)} + i\bar{h}_v^{-(Q)} v\cdot D h_v^{-(Q)}. \tag{4}$$

$h_v^{+(Q)}$ describes quarks while $h_v^{-(Q)}$ describes antiquarks. Since $m_Q \to \infty$, it requires an infinite amount of energy to create a $Q\bar{Q}$ pair, so that we need consider only heavy quarks (or antiquarks) from this point on. Note, too, that the effective Lagrangian of eqn. (4) is written for a quark traveling with a particular velocity. The full Lagrangian of HQET must therefore include a sum over all possible velocities [2], as there is an independent quark field associated with each velocity.

In addition, this Lagrangian no longer has any reference to the quark mass. Thus, if we have more than one flavor of quark for which we can take the limit $m_Q \to \infty$, the contribution to the HQET Lagrangian from each quark flavor will have a form identical to that of eqn. (4). This is the way in which the flavor symmetry becomes apparent. We emphasize that this is quite different from the flavor symmetries with which we are familiar, such as SU(2) and SU(3) symmetries for the u, d and s quarks. In the latter cases, the (approximate) symmetry exists because the quarks are (nearly) degenerate. In the case of HQET, there need not be *any* approximate degeneracy for the flavor symmetry to exist. All that is required is that for the heavy flavors, the condition $m_Q \gg \Lambda_{\rm QCD}$ must be satisfied.

The spin symmetry is also apparent from this Lagrangian, as there are no Dirac γ-matrices present. This means that at leading order, heavy quark interactions with the gluons occur only through the color-Coulomb interaction, and that interactions that can flip the spin of the heavy quark can only appear at higher order in the $1/m$ expansion.

One may think of the effective theory as one in which we expand the propagator of the heavy quark (and other operators) in powers of $1/m_Q$. Beginning with the full propagator,

$$S_Q = \frac{i(\not{P}_Q - m_Q)}{P_Q^2 - m_Q^2}, \tag{5}$$

and substituting eqn. (1) for the momentum, we find that the leading term in the propagator in the effective theory is

$$S_Q = \frac{i}{v \cdot k}. \tag{6}$$

Similar expansions for heavy quark operators must be carried out. Consider the vector current

$$V^\mu_{\rm QCD} = \bar{Q}'\gamma_\mu Q. \tag{7}$$

At leading order, and ignoring QCD radiative corrections, this becomes

$$V^\mu_{\rm HQET} = \bar{h}_{v'}^{(Q')}\gamma_\mu h_v^{(Q)}. \tag{8}$$

At order $1/m_Q$, the expansions for the heavy quark field, Lagrangian and charged vector current yield

$$Q = e^{-im_Q v\cdot x}\left[1 + \frac{i\not{D}}{2m_Q}\right] h_v^{(Q)}, \tag{9}$$

$$\mathcal{L}_v^{(Q)} = i\bar{h}_v^{(Q)} v \cdot D h_v^{(Q)} + \frac{1}{2m_Q}\bar{h}_v^{(Q)}\left[(iD)^2 - \frac{1}{2} g\sigma_{\mu\nu}G^{a\mu\nu}T^a\right] h_v^{(Q)}, \tag{10}$$

$$V^\mu_{\rm HQET} = \bar{h}_{v'}^{(Q')}\left[\gamma_\mu - \frac{1}{2m_{Q'}} i\overleftarrow{\not{D}}\right] h_v^{(Q)}. \tag{11}$$

B. Perturbative QCD

So far, we have made no mention of QCD effects. Let us now consider some operator (such as the vector current) in the full theory. In the effective theory, this operator is expanded in powers of the perturbative parameter of the effective theory, $1/m_Q$. In the full theory, such an operator will receive corrections from QCD loops. Of particular interest are loops in which the momentum of at least one of the gluons is comparable to the mass of the heavy quark. Clearly, from the way in which we have constructed the effective theory, the effects of such loops can not be reproduced in the effective theory. Recall that our starting point is eqn. (1), in which k_μ, which is a measure of the momentum of the gluons that interact with the heavy quark, is much smaller than the mass of the heavy quark. Fortunately, using the machinery of perturbative QCD and the renormalization group equations, such loops can be taken into account.

To illustrate how this works, let us sketch the steps involved in the construction of an effective theory with two heavy quarks, b and c. At energy scales μ above m_b, neither of these quarks is heavy, so we have no choice but to use full QCD. At $\mu = m_b$, we can integrate out the b quark and at scales $m_b > \mu > m_c$, for instance, we can write down the expressions above for the b quark field, Lagrangian and propagator in the effective theory. However, these expressions are valid at tree level. At one-loop level, at the energy scale $\mu < m_b$, the expression for the Lagrangian, for instance, becomes

$$\begin{aligned}\mathcal{L}_v^{(Q)} &= i\bar{h}_v^{(Q)} v \cdot D h_v^{(Q)} \\ &+ \frac{1}{2m_Q}\bar{h}_v^{(Q)}\left[a_1(\mu)(iD)^2 - \frac{a_2(\mu)}{2} g\sigma_{\mu\nu}G^{a\mu\nu}T^a\right] h_v^{(Q)},\end{aligned} \tag{12}$$

with

$$a_1(\mu) = \left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^0, \tag{13}$$

$$a_2(\mu) = \left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{-\frac{9}{33-2N_f}}. \tag{14}$$

N_f is the number of quark flavors appropriate to the momentum interval between μ and m_b. Note that both a_1 and a_2 become unity at the matching scale $\mu = m_b$, a_1 trivially so. The vector current is similarly modified if we match at the one-loop level, as an additional operator enters the picture. The new form of the charged vector current is obtained by making the replacement

$$\gamma_\mu \to \gamma_\mu - \frac{\alpha_s(m_b)}{6\pi}\gamma_\nu \not{p}\gamma_\mu \not{p}\gamma^\nu . \tag{15}$$

In addition, the current acquires a multiplicative factor

$$\left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{-\frac{6}{33-2N_f}} . \tag{16}$$

At the scale m_c, the charm quark is integrated out, and the effective theory containing heavy b and c quarks is matched onto that containing the heavy b quark, at the scale $\mu = m_c$. The charm part of the Lagrangian becomes the analog of eqn. (12). The additional complication of having two heavy quarks in the theory implies that operators that involve both flavors of heavy quark are special. For example, the new form of the charged vector current requires the replacement

$$\gamma_\mu \to \left[1 - \frac{2\alpha_s(m_c)}{3\pi} r(v\cdot v')\not{p}\right]\gamma_\mu , \tag{17}$$

with

$$r(v\cdot v') = \frac{1}{\sqrt{(v\cdot v')^2 - 1}} \ln\left[v\cdot v' + \sqrt{(v\cdot v')^2 - 1}\right] . \tag{18}$$

In addition, the vector current picks up the multiplicative factor

$$\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-\frac{6}{33-2N_f}} \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{\frac{8[v\cdot v' r(v\cdot v')-1]}{33-2N_f}} . \tag{19}$$

Thus, if we are interested in the matrix elements of some operator $\mathcal{O}$ involving heavy quarks, matching of the effective theory with the full theory means that we obtain the expression

$$\langle \mathcal{O}\rangle_{\rm QCD} = C_0 \langle \mathcal{O}_0\rangle_{\rm HQET} + \frac{C_1}{m_Q}\langle \mathcal{O}_1\rangle_{\rm HQET} \cdots . \tag{20}$$

The operators $\mathcal{O}_i$ are those that arise in the $1/m_Q$ expansion of $\mathcal{O}$, and the coefficients C_i are, in general, dependent on the scale appropriate to the expansion.

C. States

We must also redefine the heavy hadron states with which we must deal. The usual normalization of a meson state $|M(p)\rangle$ is

$$\langle M(p')|\, M(p)\rangle = 2p^0(2\pi)^3\delta^3\left(\vec{p} - \vec{p'}\right) . \tag{21}$$

We define the meson fields $\left|\tilde{M}(v)\right\rangle$ of HQET as

$$\left|\tilde{M}(v)\right\rangle = \frac{1}{\sqrt{M_M}}\,|M(p)\rangle , \tag{22}$$

so that

$$\left\langle \tilde{M}(v')\right|\left.\tilde{M}(v)\right\rangle = 2v^0(2\pi)^3\delta^3\left(\vec{p} - \vec{p'}\right) . \tag{23}$$

In the case of a baryon $\left|\tilde{\Lambda}(v,s)\right\rangle$, the normalization in HQET is

$$\left\langle \tilde{\Lambda}(v',s')\right|\left.\tilde{\Lambda}(v,s)\right\rangle = v^0(2\pi)^3\delta^3\left(\vec{p} - \vec{p'}\right)\delta_{ss'} . \tag{24}$$

In the heavy-quark limit, we note that the mass of the hadron, m_{H_Q} is

$$m_{H_Q} = m_Q + \bar{\Lambda}, \tag{25}$$

where $\bar{\Lambda}$ is determined by non-perturbative dynamics, and is of the order of a few hundred MeV. At leading order in HQET, the hadron is therefore degenerate with the heavy quark that it contains. Note that the heavy flavor symmetry implies that if there are two heavy quarks in the effective theory, then for hadrons with the same quantum numbers, $\bar{\Lambda}$ is independent of the flavor of the heavy quark. Thus, one would expect that, at leading order

$$\bar{\Lambda} = m_{H_Q} - m_Q = m_{H_{Q'}} - m_{Q'} . \tag{26}$$

III. EXCLUSIVE SEMILEPTONIC DECAYS OF MESONS

Consider the semileptonic decays $B \to D\ell\nu$ and $B \to D^*\ell\nu$, both of which proceed via the vector current $\bar{c}\gamma_\mu b$ and the axial-vector current $\bar{c}\gamma_\mu\gamma_5 b$. Using

Lorentz covariance alone, the matrix elements of these currents may be described in terms of six form factors as

$$\langle D(v')\,|\bar{c}\gamma_\mu b|\,B(v)\rangle = \sqrt{m_B m_D}\,[\xi_+(\omega)(v+v')_\mu + \xi_-(\omega)(v-v')_\mu], \quad (27)$$

$$\langle D^*(v',\epsilon)\,|\bar{c}\gamma_\mu b|\,B(v)\rangle = i\sqrt{m_B m_{D^*}}\xi_V(\omega)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta, \quad (28)$$

$$\begin{aligned}\langle D^*(v',\epsilon)\,|\bar{c}\gamma_\mu\gamma_5 b|\,B(v)\rangle &= \sqrt{m_B m_{D^*}}\,[\xi_{A_1}(\omega)(\omega+1)\epsilon^*_\mu \\ &\quad - \xi_{A_2}(\omega)(\epsilon^*\cdot v)v_\mu - \xi_{A_3}(\omega)(\epsilon^*\cdot v)v'_\mu]\,, \end{aligned}\quad (29)$$

where $\omega = v\cdot v'$.

These form factors represent all our ignorance of non-perturbative QCD effects in the binding and interactions of the mesons. They are unknown functions of the kinematic invariant $q^2 = (m_B v - m_{D^{(*)}}v')^2$. Furthermore, the normalizations of these form factors are unknown. Thus, if we are to use the measured decay rates of the above processes to extract V_{cb}, some set of *ansätze* have to be made for the ξ_i.

If we now apply the symmetries of HQET, we find that all six form factors may be written in terms of a single form factor [1], ξ, with

$$\xi_- = \xi_{A_2} = 0, \quad (30)$$

$$\xi_+ = \xi_V = \xi_{A_1} = \xi_{A_3} \equiv \xi. \quad (31)$$

The kinematic dependence of ξ on ω is still unknown. However, the symmetries of HQET also allow us to absolutely normalize this form factor at the so-called non-recoil point, $v = v'$. There, $\xi(\omega)|_{\omega=1} = 1$, where $\omega = v\cdot v'$ is the new kinematic variable. ξ is the so-called Isgur-Wise (IW) function.

ξ is a universal form factor. This means that for any pseudoscalar meson consisting of a heavy quark and a light antiquark (or vice versa) decaying into another heavy pseudoscalar or its companion vector meson, via a heavy-quark current, the IW function is the same as the one above. This is a consequence of the flavor symmetry discussed in the previous section. We note also that the IW function would be the same for processes mediated by heavy-quark currents with different Lorentz and Dirac structure.

The normalization of the IW function at the non-recoil point is easy to understand intuitively. Since the heavy quark acts as a static source of color, all of the QCD dynamics involved in the form factor comes from the brown muck. In fact, the IW function is just the overlap of the appropriately boosted initial and final state wave functions of the brown muck. At the non-recoil point, where both initial and final wave functions are boosted identically, this overlap is unity.

Now that we know the normalization of the IW function, we can examine the quantity

$$\lim_{\omega\to 1}\frac{1}{\sqrt{(\omega)^2-1}}\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3}\left(m_B - m_{D^*}\right)^2 m_{D^*}^3 \times |V_{cb}|^2\,|\xi_{A_1}(\omega=1)|^2 \quad (32)$$

in $B\to D^*\ell\nu$. In this expression, everything on the right-hand-side, with the exception of $|V_{cb}|$ is known. Thus, this offers a means of extracting $|V_{cb}|$ from measurements of the differential decay rate, in a model-independent way [5].

However, if this quantity is to be determined with precision, we must take into account that c and b quarks have *finite* masses. This is done by including $1/m$ corrections. Since $m_c \approx 1.5$ GeV, and $\bar{\Lambda}/m_c \approx 0.3$, these corrections are potentially large. In addition, QCD radiative corrections should be included. In principle, all of this will complicate matters, as one would expect that new, uncalculable and unnormalizable form factors will enter the picture. This is in fact the case, in general [6]. However, at the so-called non-recoil point (ie., at $v = v'$), the $1/m_c$ and $1/m_b$ corrections to the normalization of the IW function vanish, a consequence of Luke's theorem [6]. Thus, the leading power corrections are of order $1/m_c^2$.

The corrected value of $\xi_{A_1}(1)$ is

$$\xi_{A_1}(1) = \eta_A + \delta_{m_Q^2}, \quad (33)$$

with

$$\begin{aligned}\eta_A(1) &= x^{6/25}\left[1 + 1.561\frac{\alpha_s(m_c)-\alpha_s(m_b)}{\pi} - \frac{8\alpha_s(m_c)}{3\pi}\right.\\ &\quad + z\left\{\frac{25}{54} - \frac{14}{27}x^{-9/25} + \frac{1}{18}x^{-12/25} + \frac{8}{25}\ln x\right\}\\ &\quad \left. - \frac{\alpha_s(\bar{m})}{\pi}\frac{z^2}{1-z}\ln z\right], \\ x &= \frac{\alpha_s(m_c)}{\alpha_s(m_b)}, \qquad z = \frac{m_c}{m_b}, \qquad m_c < \bar{m} < m_b.\end{aligned} \quad (34)$$

The three terms above arise from radiative corrections [3,4]. The first of these is due to running between m_b and m_c, while the second arises from integrating out the b quark at the scale m_b, matching the effective theory to full QCD at order $1/m_b$, running down to m_c, integrating out the c quark, and matching at the scale m_c. The third term arises from following a similar procedure, but integrating out the b and c quarks simultaneously at some scale $\bar{m}$. This gives the full z dependence, but at the cost of a scale ambiguity.

Note that there are no $1/m$ corrections, which is a consequence of Luke's theorem. The $1/m^2$ corrections have the form [7]

$$\delta_{m_Q^2} = -\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)\left(\frac{\ell_V}{2m_c} - \frac{\ell_P}{2m_b}\right) + \frac{\Delta}{4m_c m_b}. \tag{35}$$

The coefficients $\ell_{P,V}$ are related to matrix elements of the vector current at zero recoil,

$$\langle D(v)\,|\bar{c}\gamma_\mu b|\,B(v)\rangle = 2\eta_V v_\mu \left[1 - \ell_P\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 + \ldots\right], \tag{36}$$

$$\langle D^*(v,\varepsilon)\,|\bar{c}\gamma_\mu b|\,B(v,\varepsilon)\rangle = 2\eta_V v_\mu \left[1 - \ell_V\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 + \ldots\right], \tag{37}$$

with

$$\eta_V = \eta_A\left[1 + \frac{2\alpha_s(m_b)}{3\pi} - \frac{14}{27}z\left(1 - 4x^{-9/25} + 3x^{-12/25}\right)\right]. \tag{38}$$

The coefficient Δ may be written

$$\Delta = \frac{1}{2}\left(m_V^2 - m_P^2\right) + \frac{4}{3}\lambda_1 + \ldots, \tag{39}$$

where the parameter λ_1 is defined by

$$\left\langle B(v)\left|\bar{h}_v^{(b)}(iD)^2 h_v^{(b)}\right| B(v)\right\rangle = 2\lambda_1 M_B. \tag{40}$$

The $1/m^2$ corrections have been estimated to be anywhere between 3% and −8%.

Experimentally, the quantity on the left-hand-sidè of eqn. (32) is obtained by measuring the decay rate as close as possible to the kinematic end-point, and extrapolating to the kinematic end-point. The process of extrapolation introduces some uncertainty, as a specific form for the IW function must be assumed. Figure 1 shows the results from the CLEO collaboration [8], using two different kinds of extrapolation,

$$\xi(\omega) = 1 - a^2(\omega - 1) + b(\omega - 1)^2, \tag{41}$$

to fit the data. Fits are performed requiring $b = 0$ (linear fit) and allowing b to float (quadratic fit). With the quadratic fit, large statistical errors on the parameters a and b are obtained, indicating that the data are not sufficient to discriminate between the two forms for $\xi(\omega)$.

The result obtained by CLEO from the various fits is

$$|V_{cb}|\xi_{A_1} = 0.0351 \pm 0.0019\,(\text{stat.}) \pm 0.0018\,(\text{sys.}) \pm 0.0008\,(\text{lifetime}). \tag{42}$$

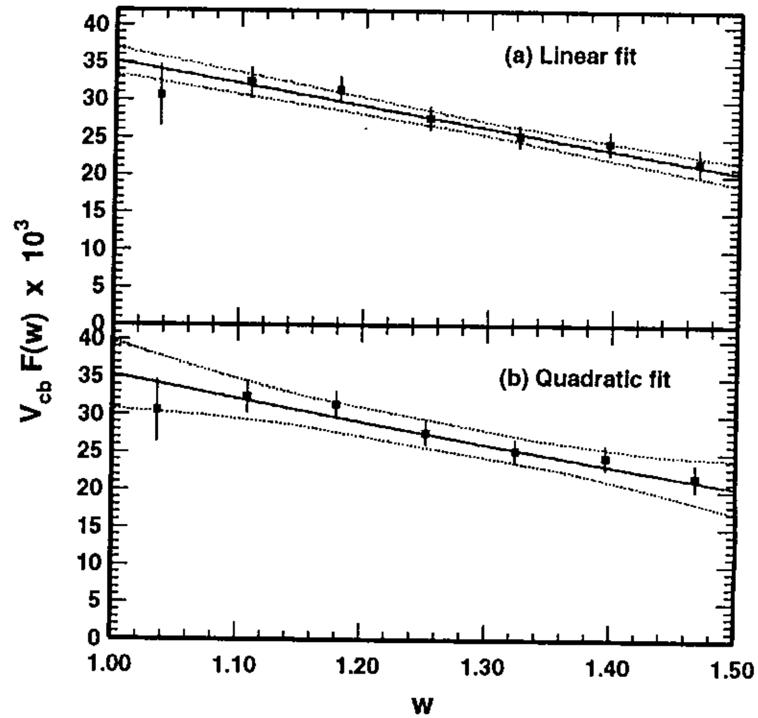

FIG. 1. The product $|V_{cb}|\xi_{A_1}(y)$ as determined in the CLEO fits. The dotted lines show the contours for 1σ variations of the fit parameters.

Using Neubert's estimate for the normalization of ξ_{A_1} [9], they find

$$|V_{cb}| = 0.0362 \pm 0.0019\,(\text{stat.}) \pm 0.0020\,(\text{sys.}) \pm 0.0014\,(\text{model}), \tag{43}$$

where the last uncertainty arises from the parametrization of the IW function, and the uncertainty in the lifetime has been included in the systematic error for $|V_{cb}|$.

IV. INCLUSIVE DECAYS

The formalism outlined above may be easily extended to other semileptonic decays of B mesons, such as decays to excited D^{**} resonances [10]. In fact, experimental observations indicate that decays to such inelastic channels are important in explaining the observed inclusive semileptonic decay rate of the B meson, as the elastic channels (D, D^*) account for only about 60% of the inclusive semileptonic rate [11].

The inclusive decays have been treated in two ways. In one approach, the inclusive rate may be written as the sum of decay rates into exclusive final states. Within the framework of HQET, the decays to excited states are easily treated. The lowest lying excited states are expected to be the $j^P = 1/2^+$ (0^+, 1^+) and $j^P = 3/2^+$ (1^+, 2^+) multiplets, corresponding to the P-wave mesons. The semileptonic decays of the B meson to the states of each such multiplet are described in terms of a single form factor, analogous to the IW function for the elastic channels [10,12]. As with the elastic IW function, these form factors are universal and uncalculable. Unlike the elastic IW function, their normalizations are unknown.

By expressing the rate for the inclusive semileptonic decay as a sum over exclusive final states [13], the sum rule

$$1 = \left(\frac{\omega+1}{2}\right)|\xi(\omega)|^2 + (\omega-1)\sum_n\left[\left(\frac{\omega^2-1}{2}\right)\left|\xi^{(n)}(\omega)\right|^2 + 2\left|\xi^{(n)}_{1/2}(\omega)\right|^2(\omega+1)^2\left|\xi^{(n)}_{3/2}(\omega)\right|^2\right] + \ldots \tag{44}$$

is obtained [14]. $\xi^{(n)}(\omega)$ are the IW functions for decays into the radial excitations of the ground state pseudoscalar and vector mesons. The ellipsis denotes contributions from other resonances as well as from the continuum. By the continuum we mean contributions from processes like $B \to D\pi\ell\nu$, where the pion does *not* result from the decay of a D^{**} resonance. Note that this sum rule is valid throughout all of the kinematically allowed region.

If we now focus on the region near the non-recoil point, and expand all of the IW functions in Taylor series around $v \cdot v' = 1$,

$$\xi(\omega) = \xi(1) - (\omega-1)\xi'(1) + \ldots, \tag{45}$$

ve obtain the relation

$$\xi'(1) = \frac{1}{4} + \sum_n\left[\left|\xi^{(n)}_{1/2}(1)\right|^2 + \left|\xi^{(n)}_{3/2}(1)\right|^2\right] + \ldots > 1/4, \tag{46}$$

·here the last inequality results since the terms in the summations are all ositive-definite. Thus we have a limit on the slope of the IW function. The ıll relation above (eqn. (44)) can also be used to extract constraints on the IW ınction itself. Neglecting all but the first term on the RHS leads to

$$\xi(\omega) \le \sqrt{\frac{2}{\omega+1}}, \tag{47}$$

which, however, is not very restrictive. De Rafael and Taron [15], along with other authors [16], have used the analytic structure of $\xi(\omega)$ in the complex-ω plane, together with a perturbative calculation, to extract more stringent constraints on the IW function. Eqn. (44) has also been used to test models of the IW function [17].

Alternatively, the inclusive decay rate may be treated in the operator product expansion (OPE) [18,19]. Let us consider the decays $B \to X\ell\nu$, where X may or may not be charmed. The inclusive decay rate may be written in terms of the hadronic tensor $W_{\mu\nu}(q,v)$, where

$$W_{\mu\nu}(q,v) = \sum_X \int (2\pi)^4\delta^4(P_B - q - P_X) < B(v)|(\bar{b}\gamma_\mu(1-\gamma_5)q)|X >< X|(\bar{q}\gamma_\nu(1-\gamma_5)b)|B(v)> . \tag{48}$$

If the leptonic spins are unobserved, the lepton tensor is

$$\Lambda_{\mu\nu} = 8\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(kk') + i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right), \tag{49}$$

and the differential decay rate in the rest frame of the decaying B meson is

$$d\Gamma = \frac{G_F^2}{4m_B}|V_{bq}|^2 W_{\mu\nu}(k+k',v)\Lambda^{\mu\nu}d(PS), \tag{50}$$

where $d(PS)$ is the phase space integral.

Using standard techniques, the hadronic tensor is rewritten

$$W_{\mu\nu}(q,v) = \int d^4x e^{iqx} \times < B(v)|(\bar{b}(x)\gamma_\mu(1-\gamma_5)q(x))(\bar{q}(0)\gamma_\nu(1-\gamma_5)b(0))|B(v)> . \tag{51}$$

This hadronic tensor may be described in terms of a set of scalar form factors. We now consider the time ordered product

$$T_{\mu\nu}(q,v) = \int d^4x e^{iqx} \times < B(v)|T\left[(\bar{b}(x)\gamma_\mu(1-\gamma_5)q(x))(\bar{q}(0)\gamma_\mu(1-\gamma_5)b(0))\right]|B(v)>, \tag{52}$$

which may also be written in terms of a set of scalar form factors. The form factors of $W_{\mu\nu}$ may be related to the discontinuities of the form factors of $T_{\mu\nu}$, and it is the object $T_{\mu\nu}$ that is treated in the OPE in powers of $1/m_b$.

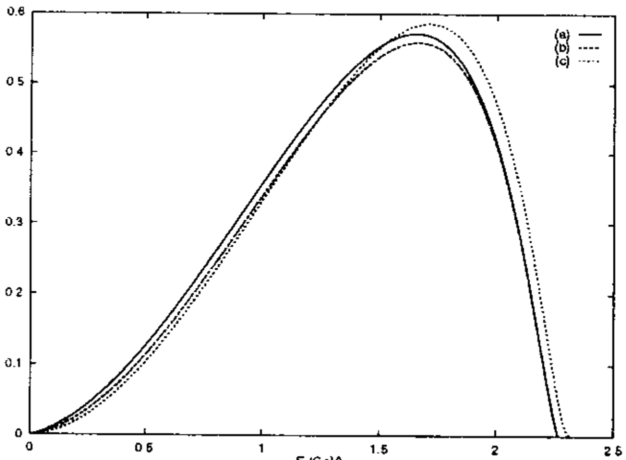

FIG. 2. The lepton energy spectrum $d\Gamma/dE \times 10^{15}$ of the inclusive semileptonic decay $\overline{B}^0 \to X_c e^- \bar{\nu}_e$. The curves are: (a) full result including long- and short distance contributions; (b) Result without short distance QCD corrections; (c) leading order result, i.e. the parton model result.

The leading term in this OPE corresponds to the semileptonic decay of a free b quark. HQET and the OPE allow corrections to this limit to be systematically included, and the inclusive lepton spectrum that results from the calculation of Mannel [19] is shown in figure 2.

V. NONLEPTONIC DECAYS

The nonleptonic weak decays of hadrons are very difficult to understand. The factorization hypothesis has been used in studying some of the nonleptonic decays of mesons. The work of ref. [20] has gone some way in establishing the usefulness of this approximation from a purely phenomenological standpoint. In contrast with this is the formal basis of the approximation which, until the advent of HQET, has relied mainly on large N_c arguments where N_c is the number of colors. As is well known, such arguments are not always trustworthy phenomenologically.

These arguments have been elaborated in the work of Bardeen, Buras and Gérard [21]. Using an effective meson theory that reflects the large N_c limit of QCD, they argue that matrix elements for nonleptonic decays do indeed factorize, at least to lowest order in $1/N_c$. They also discuss next-to-leading order contributions in the $1/N_c$ expansion, as well as QCD scaling of operators to order $1/N_c$.

As an example of the application of the factorization hypothesis, let us consider the decay $\bar{B}^0 \to \bar{D}^+\pi^-$. One assumes that

$$< \bar{D}\pi|\mathcal{O}_1|B > \approx < \bar{D}|j_\mu|B >< \pi|j^\mu|0 >, \tag{53}$$

where $j_\mu = \bar{q}\Gamma_\mu q' = \bar{q}\gamma_\mu(1-\gamma_5)q'$ is the left handed current, and

$$\mathcal{O}_1 = \bar{b}\Gamma_\mu c\bar{u}\Gamma^\mu d. \tag{54}$$

$\mathcal{O}_1$ is a term in the effective low-energy Hamiltonian for weak interactions, $\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}V^*_{ud}\mathcal{O}_1 + \ldots$. When QCD effects are taken into account, mixing between this operator, which is of dimension 6, and other dimension 6 operators, is induced. For instance, at some new energy scale, the operator

$$\mathcal{O}_2 = \bar{u}j^\mu c\bar{b}j_\mu d, \tag{55}$$

will play a role.

An intuitive justification of factorization comes from color transparency arguments [22]. These arguments have been used to explain features of nuclear scattering experiments. They assume that only the valence Fock component of a hadron's wave function with small transverse size of order $1/Q$ (Q is the momentum transfer in the process) contributes to an exclusive amplitude at high momentum transfer. This component of the wave function has a small color dipole moment and thus has a strong-interaction cross section of order $1/Q^2$. For the nonleptonic weak decays, this means that an energetic color singlet pair of quarks produced from a virtual W will propagate through the surrounding hadronic matter essentially unperturbed: to leading order in $1/Q$, the amplitudes for nonleptonic decays factorize.

More recently, Dugan and Grinstein [23] have formulated a QCD basis for factorization in the decays of heavy hadrons. Their argument is limited to a specific class of decays, in which the pair of quarks produced from the virtual W are very energetic and highly collimated. Using the framework of HQET, coupled with a 'high-energy' expansion, they show rigorously that the matrix elements of the operator $\mathcal{O}_2$ are suppressed by powers of $1/m_b$, $1/m_c$ or $1/E$, where E is the total energy of the light quark pair produced from the virtual W. This means that to leading order in $1/E$, factorization holds. They proceed further in evaluating the logarithmic corrections to the coefficient of $\mathcal{O}_1$. These corrections lead to a final form

$$
\begin{aligned}
\langle \bar{D}\pi | \mathcal{H}_{eff} | B \rangle \approx & \left[\frac{1}{3} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{-12/23} \left(\frac{\alpha'_s(m_b)}{\alpha'_s(E)} \right)^{-12/25} \right. \\
& \left. + \frac{2}{3} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{6/23} \left(\frac{\alpha'_s(m_b)}{\alpha'_s(E)} \right)^{-3/25} \right] \left(\frac{\alpha'_s(E)}{\alpha'_s(m_c)} \right)^{a_I} \\
& \times \left(\frac{\alpha''_s(m_c)}{\alpha''_s(\mu)} \right)^{a_L} \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} \langle \bar{D} | j_\mu | B \rangle \langle \pi | j^\mu | 0 \rangle , \qquad (56)
\end{aligned}
$$

valid if $E >> m_c$, while for $m_c >> E$, the expression is

$$
\begin{aligned}
\langle \bar{D}\pi | \mathcal{H}_{eff} | B \rangle \approx & \left[\frac{1}{3} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{-12/23} \left(\frac{\alpha'_s(m_b)}{\alpha'_s(m_c)} \right)^{-12/25} \right. \\
& \left. + \frac{2}{3} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{6/23} \left(\frac{\alpha'_s(m_b)}{\alpha'_s(m_c)} \right)^{-3/25} \right] \\
& \times \left(\frac{\alpha''_s(m_c)}{\alpha''_s(\mu)} \right)^{a_L} \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} \langle \bar{D} | j_\mu | B \rangle \langle \pi | j^\mu | 0 \rangle . \qquad (57)
\end{aligned}
$$

α_s, α'_s and α''_s are the values appropriate to a theory with 5, 4 and 3 flavors of quarks, respectively, and

$$
a_I = -6/25; \qquad a_L = \frac{8}{27}(v \cdot v' r(v \cdot v') - 1), \qquad (58)
$$

with

$$
r(v \cdot v') = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ln \left(\sqrt{(v \cdot v')^2 - 1} + v \cdot v' \right) . \qquad (59)
$$

μ is a low energy scale at which the dynamics is described by HQET. Note that the heavy quark current in eqn. (56) corresponds to the effective current of HQET.

Application of factorization to the two-body nonleptonic decays of B mesons, for instance, leads to predictions such as

$$
\frac{\Gamma(\bar{B} \to D\rho^-)}{\Gamma(\bar{B} \to D\pi^-)} = \left(\frac{f_\rho}{f_\pi} \right)^2 , \quad \frac{\Gamma(\bar{B} \to D\pi^-)}{\Gamma(\bar{B} \to D^*\pi^-)} = 1. \qquad (60)
$$

Experimental results are more or less consistent with these predictions, so far [24].

VI. $\Lambda_B \to \Lambda_C$ EXCLUSIVE SEMILEPTONIC DECAYS

Consider $\Lambda_b \to \Lambda_c \ell \nu$. By Lorentz covariance alone, the matrix elements of the left handed current are written as

$$
\begin{aligned}
\langle \Lambda_c(p') | \bar{c}\gamma_\mu (1 - \gamma_5) b | \Lambda_b(p) \rangle = & \bar{u}(p') \left[(f_1 \gamma_\mu + i f_2 \sigma_{\mu\nu} q^\nu + f_3 q_\mu) \right. \\
& \left. - (g_1 \gamma_\mu + i g_2 \sigma_{\mu\nu} q^\nu + g_3 q_\mu) \gamma_5 \right] u(p). \qquad (61)
\end{aligned}
$$

Using the symmetries of HQET, at leading order, we find [25–27]

$$
\langle \Lambda_c(v') | \bar{c}\gamma_\mu (1 - \gamma_5) b | \Lambda_b(v) \rangle = \bar{u}(v') \gamma_\mu (1 - \gamma_5) u(v) \eta(\omega), \qquad (62)
$$

where $\eta(\omega)$ is the baryonic version of the IW function. As with the mesons, it is a universal, uncalculable function of ω. In addition, this form factor is also normalized at the non-recoil point, with $\eta(\omega)|_{\omega=1}$. If we compare eqns. (61) and (62), we find that

$$
f_1 = g_1 = \eta, \qquad (63)
$$

$$
f_2 = g_2 = f_3 = g_3 = 0. \qquad (64)
$$

The first of these equations implies that $G_A = G_V$ for these decays.

As with B decays, we can write

$$
\lim_{\omega \to 1} \frac{1}{\sqrt{(\omega)^2 - 1}} \frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3} \left(m_{\Lambda_b} - m_{\Lambda_c} \right)^2 m^3_{\Lambda_c} |V_{cb}|^2 |\eta(\omega = 1)|^2 . \qquad (65)
$$

Everything on the right-hand-side, except for $|V_{cb}|$, is again known. Thus, observation of the elastic semileptonic decays of the Λ_b will lead to a second, independent, model-independent determination of $|V_{cb}|$. As with B decays, there are radiative and power corrections to $\eta(1)$. Luke's theorem again means that the first power corrections appear at $\mathcal{O}(1/m^2)$.

For the nonleptonic decay $\Lambda_b \to \Lambda_c \pi$, the decay rate may be written

$$
\begin{aligned}
\Gamma = \Gamma_0 & \left[1 + \alpha \left(\hat{S}_{\Lambda_b} + \hat{S}_{\Lambda_c} \right) \cdot \hat{P} - \beta \hat{P} \cdot \left(\hat{S}_{\Lambda_c} \times \hat{S}_{\Lambda_b} \right) \right. \\
& \left. + (1 - \gamma) \hat{S}_{\Lambda_b} \cdot \hat{P} \hat{S}_{\Lambda_c} \cdot \hat{P} + \gamma \hat{S}_{\Lambda_b} \cdot \hat{S}_{\Lambda_c} \right] , \qquad (66)
\end{aligned}
$$

where $\hat{S}_{\Lambda_b}$ and $\hat{S}_{\Lambda_c}$ are unit vectors in the directions of polarization of the Λ_b and Λ_c, respectively, and $\hat{P}$ is a unit vector in direction of motion of the Λ_c, in the rest frame of the parent Λ_b. The predictions of HQET coupled with factorization are that $\alpha = -1$ and $\beta = \gamma = 0$ [28].

VII. $\Lambda_c \to \Lambda$ EXCLUSIVE SEMILEPTONIC DECAYS

HQET may also be applied to Λ_c semileptonic decays, where it is found that [27]

$$\begin{aligned}&\langle \Lambda(p') | \bar{s}\gamma_\mu (1-\gamma_5) c | \Lambda_c(v)\rangle \\ &= \bar{u}(p') \left[F_1(v\cdot p') + \not{v} F_2(v \cdot p')\right] \gamma_\mu (1-\gamma_5) u(v). \end{aligned} \tag{67}$$

For these decays, the normalization of neither F_1 nor F_2 is known. This is because we no longer have a symmetry between the initial and final states: the s quark in the final Λ is not being treated as a heavy quark, so that the flavor symmetry discussed in the preliminaries no longer exists.

Nevertheless, the form above yields $G_A = G_V$, and

$$f_1 = g_1 = F_1 + \frac{m_{\Lambda_c}}{m_{\Lambda_b}} F_2, \tag{68}$$

$$f_2 = f_3 = g_2 = g_3 = \frac{1}{m_{\Lambda_b}} F_2. \tag{69}$$

Note that this form is valid for the semileptonic decay of *any* heavy Λ_Q to *any* $J^P = 1/2^+$ baryon, such as $\Lambda_c \to n\ell\nu$ or $\Lambda_b \to p\ell\nu$. The form above leads to the prediction $\alpha_{\Lambda_c} = -1$, where α_{Λ_c} is a polarization variable appropriate to the semileptonic decay of the Λ_c. ARGUS has found that $\alpha_{\Lambda_c} = -0.91 \pm 0.49$ in $\Lambda_c \to \Lambda\ell\nu$, while CLEO has found $\alpha_{\Lambda_c} = -0.89^{+0.17+0.09}_{-0.11-0.05}$ [29], both consistent with the prediction of HQET.

As with the Λ_b's, we may employ the factorization assumption in applying HQET to the process $\Lambda_c \to \Lambda\pi$. The predictions there are that $\alpha \approx -1$, $\beta = \gamma \approx 0$. This is quite a striking prediction, as we do not know the normalizations of the form factors F_1 and F_2. Nevertheless, this prediction can be made, independent of these unknown normalizations. Experimentally, ARGUS finds $\alpha = -0.96 \pm 0.42$, while CLEO has measured $\alpha = -1.1 \pm 0.4$ [30], both consistent with the prediction.

Consideration of the Λ_c semileptonic decays leads to a very important implication for $\Lambda_b \to \Lambda_c$ semileptonic decays. In the case of the latter, we expect the leading corrections to all predictions to be $\mathcal{O}(\bar{\Lambda}/m_c) \approx 30\%$, which is significant. Let us now imagine treating the charm quark as a light quark. Then, the results of eqns. (67) - (69) must hold. Now, we revert to treating charm as heavy, and perform the expansion in $1/m_c$. The results of eqns. (67) - (69) must still hold, since these result from an all-orders calculation in $1/m_c$, in some sense. This means that the predictions $f_1 = g_1$, $\alpha_{\Lambda_b} = -1$ etc., are valid independent of $1/m_c$ corrections, and will hold to *all* orders in the $1/m_c$ expansion [31]. Furthermore, these predictions will receive no radiative corrections that are proportional to powers of $\alpha_s(m_c)$, as the results of eqns. (67) - (69) do not receive such corrections (since the 'light' charm quark is not integrated out).

VIII. MISCELLANY

The subjects discussed above are but appetizers from the smorgasbord of topics treated in HQET. Space limitations do not allow us to treat any of the remaining topics in a similar fashion, which was already quite cursory. To give the reader a flavor (or taste) of what else has been done in HQET, we conclude with a sampling (somewhat random) of subjects.

A. Strong Decays

Consideration of the symmetry structure of the HQET multiplets allows us to infer relationships among the decay rates for the strong decays of the states of one multiplet to those of another, with emission of a common daughter hadron such as a pion. For instance, let us consider $D^{**} \to D\pi$, where D^{**} denotes the states of the $j^P = 1/2^+$ $(0^+, 1^+)$ multiplet. The four decays $0^+ \to D\pi$, $0^+ \to D^*\pi$, $1^+ \to D\pi$ and $1^+ \to D^*\pi$ are described in terms of a single unknown constant, so that ratios of the decay amplitudes can be predicted [32]. We can also deduce that the states of the $j^P = 1/2^+$ multiplet will be broader than those of the $j^P = 3/2^+$ multiplet.

These results may also be obtained in the framework of chiral perturbation theory (CPT) [33–35]. In the marriage of HQET and CPT, the transformations of the multiplets of degenerate heavy hadrons under chiral rotations is easily specified, so that couplings to pions, for instance, can be treated.

B. $B \to D^{(*)}\pi\ell\nu$

The inelastic channels apparently contribute significantly to the inclusive semileptonic decays of the B meson [11]. Perhaps the simplest inelastic channel to treat theoretically is $B \to D^{(*)}\pi\ell\nu$, where the pion may or may not result from the decay of D^{**} resonance. This process has been discussed by a number of authors [36–39]. In all of these articles, the theoretical framework is the combination of HQET and CPT, so that pions in the decay are soft. The authors of refs. [36]- [38] have considered the contributions arising from figure 3, along

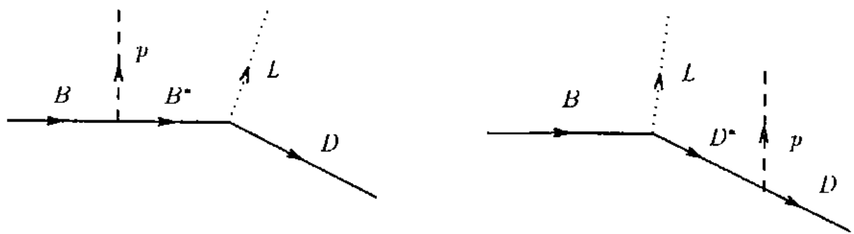

FIG. 3. Diagrams contributing to $B \to D\pi\ell\bar{\nu}$. The dashed line represents the pion and the dotted line denotes the virtual W.

with similar diagrams containing a D^* in the final state, while the authors of ref. [39] have included diagrams in which the intermediate states may be from any of the $(0^-, 1^-)$, $(0^+, 1^+)$, $(1^-, 2^-)$ or $(0^-, 1^-)'$ multiplets. The states in the last multiplet are the first radial excitations of the ground states. The results from all of these analyses indicate that the contributions from the inelastic channels considered are of the order of 1% of the decay width of the B meson.

C. Meson Decay Constants

The charged pseudoscalar and vector mesons may decay purely leptonically ($B^{(*)-} \to \ell\bar{\nu}$, for example), and the matrix elements for the decays are

$$\langle 0 | \bar{q}\gamma_\mu (1-\gamma_5) b | B(v) \rangle = f_B m_B v_\mu, \tag{70}$$

$$\langle 0 | \bar{q}\gamma_\mu (1-\gamma_5) b | B^*(v,\varepsilon) \rangle = f_{B^*} m_{B^*} \varepsilon_\mu. \tag{71}$$

with similar expressions for the charmed mesons. The meson decay constants f_B and f_{B^*} are important parameters for the calculation of $B-\bar{B}$ mixing amplitudes. Leading order HQET predicts that

$$\frac{f_{B^*}}{f_B} = 1, \tag{72}$$

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}. \tag{73}$$

Including radiative corrections modifies the leading order predictions to [9]

$$\frac{f_{B^*}}{f_B} = 1 - \frac{2\alpha_s(m_b)}{3\pi}, \tag{74}$$

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{\frac{6}{25}} \left[1 + 0.894\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right] \approx 0.69. \tag{75}$$

Power ($1/m_Q$) corrections can also be systematically included [9].

D. Reparametrization Invariance

The relation

$$P_Q^\mu = m_Q v^\mu + k^\mu \tag{76}$$

for the momentum of the quark is one of the key points in the development of HQET. In this expression, the velocity of the quark inside the hadron has been identified with that of the hadron itself. However, there is no reason why some other velocity, $w \equiv v + q/m_Q$, say could not be chosen. Here, q is of the order of $\bar{\Lambda}$, and is subject to the constraint $2v \cdot q + q^2/m_q = 0$. This ensures that $v^2 = w^2 = 1$. We can therefore write the momentum of the heavy quark in two equivalent ways as

$$P_Q^\mu = m_Q v^\mu + k^\mu = m_Q w^\mu + k'^\mu, \tag{77}$$

with $k' = k - q$. The effective theories constructed using v and w must lead to the same results. In other words, the effective Lagrangian must be invariant under the reparametrization of eqn. (77). The implications of this concept have been explored [40]. One profound consequence is that in the heavy mass expansion, one can find relationships among the coefficients of operators that arise at different orders in $1/m_Q$.

E. Miscellaneous Miscellany

There are many more aspects of HQET that have not been discussed. We conclude by running through a list of topics which space does not allow us to discuss, but only to mention. In addition to the sum rule obtained by Bjorken and collaborators, there is the optical sum rule obtained by Voloshin [41], as well as the Mössbauer type sum rule of Lipkin [42]. The inclusive semileptonic decays have been treated by a number of authors [43]. Using the operator product expansion, the authors of ref. [44] have suggested that the inclusive semileptonic decays can be used for a model independent determination of $|V_{ub}|$.

HQET has also had much interplay with many other areas of hadron spectroscopy, including lattice calculations, relativistic and nonrelativistic quark models, and QCD sum rules. All of these approaches have been used to calculate the properties of heavy mesons such as their masses, decay constants and form factors [9]. The rare and nonleptonic decays of heavy hadrons, the fragmentation, hadronization and polarization of heavy quarks produced in collider experiments, and the use of heavy quarks to test the standard model and seek new physics have all received some attention in the literature. Both experimentally and theoretically, much is being done to test the predictions of HQET, and to delimit its range of applicability. It is safe to say that this field will remain very active for some time to come.

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